Chapter 8

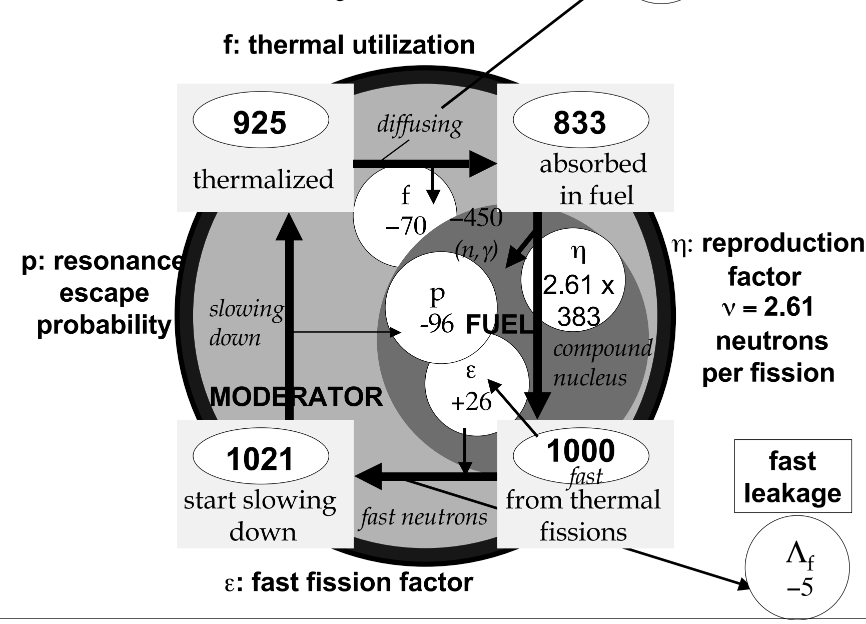
# Power Changes in a Critical Reactor

- ***For very small reactivity increases***
- ***For small reactivity increases***
- ***For large reactivity increases/decreases***

## The Critical Reactor

- **$k = \eta f p \varepsilon \Lambda_f \Lambda_t = 1$**
  - $\eta$ is the reproduction factor, 1.20030
  - f is the thermal utilization, 0.92248
  - p is resonance escape probability, 0.90634
  - $\varepsilon$ is the fast fission factor, 1.02597
  - $\Lambda_f$ is the fast non-leakage probability, 0.9951
  - $\Lambda_t$ is the thermal non-leakage probability, 0.9762
    - $\eta$,p & $\varepsilon$ are design values for burnup 1.66 n/kb
    - $\Lambda_f \Lambda_t$ agree with design estimates of leakage
    - f is chosen to give k = 1

# Neutron Cycle




# Parasitic (Non-Fuel) Absorption

- 26 in zirconium (pressure tubes etc.)
- 22 in heavy water
- 3 in instruments and structures
- 16 in adjusters
- $\boxed{\text{3 in liquid zone control compartments}}$

70 in Total (typical values)

**By adjusting the water ($H_2O$) level in the zone compartments we can make k slightly bigger or smaller than 1.**

## Reactivity

- Notation: we use $\Delta k = k - 1$
- To measure how far the reactor is from critical ($k = 1$) we can use
  - $\Delta k$ the absolute difference
  - $\Delta k/k$ the relative difference $= \rho$
- Our notes call $\Delta k$ the reactivity
- Most textbooks call $\Delta k/k$ the reactivity
- When k is nearly = 1 there is no numerical difference.

## Neutron Cycles for k > 1 Initial Power is $P_0$

- At time $t = 0$ add small $+ \Delta k$

1st generation $P_1 = k\, P_0$

2nd generation $P_2 = k\, P_1 = k^2 P_0$

3rd generation $P_3 = k\, P_2 = k^3 P_0$

•

after n cycles $P_n = k^n P_0$

## Neutron Lifetime and Reactor Period

- $\mathbf{P_n = k^n P_0}$ is a perfectly good formula
- But, it is useful to write the formula as an exponent in terms of:
  - time, t, rather than number of cycles, n.
    - t is time after $\Delta k$ is added
  - *reactor period*, $\tau$
    - a parameter to characterize rate of power increase
  - *neutron lifetime*, $\mathcal{L}$
    - average time of one cycle

$$\boxed{\mathbf{P(t) = P_0\, e^{t/\tau}}}$$

## Playing Games with the Math

$P_n \quad = k^n P_0$ can be written as

$P(t) = P_0\, e^{t/\tau}$

if $k^n = e^{t/\tau}$ or $n \supseteq ln(1 + \Delta k) = t/\tau$

$ln(1 + \Delta k) = \Delta k$ for very small $\Delta k$, and,

$t = n \times \mathcal{L}$ (by definition of $\mathcal{L}$) so

$$\boxed{\tau = \mathcal{L}/\Delta k}$$

# More Math
(This is how the text does it.)

- We get the same result if we set the average rate of change in neutron population over one cycle equal to the instantaneous rate of change
- $(n \text{ x } \Delta k)$ $\mathcal{L}$ is average rate of change
  - dn/dt is the instantaneous rate of change
- $(1/n)[dn/dt] = \Delta k/\mathcal{L} = 1/\tau$, so
- $\ln(n) = t/\tau + \ln(n_0)$
- $n(t) = n_0 e^{t/\tau}$ or, $P(t) = P_0 e^{t/\tau}$
- since power from fission is $\propto n$

# Some Hidden Assumptions

- Control systems add positive reactivity in a sequence of small steps; a ramp, not a single large step.
  - In Ch 13 we will talk more about control.
- When temperatures change, $\eta$, f, p & $\Lambda$ change. These changes are small at low power (below, say, 5% full power).
  - A reactivity change will not remain constant
  - Ch 12 explains how temperature affects reactivity

## More Hidden Assumptions

- A change in fission rate changes the rates of production and burnout of fission products. Neutrons absorption changes so reactivity changes.
- Again, reactivity will not be constant
  - Chapter 11 is about major fission product effects.
- Reactor thermal power comes directly from fission, but also from fission product decay. Our formulas are for “neutron power”
  - Chapter 10 compares thermal power, neutron power and total fission power.

## What is the Neutron Lifetime?

- Most neutrons take about 1 ms ($10^{-3}$ s), from the moment of fission, to slow down, diffuse, be absorbed and cause a fission.
  - *Prompt Neutron Lifetime* $\ell$ ♠ $10^{-3}$ s (CANDU)
  - In a PWR $\ell$ is about 0.3 ms
- About ›% of the neutrons from fission come after β decay of the fission products.
  - Delayed neutrons arrive a few tenths of a second later, up to several minutes later.

## Neutron Lifetime - First Guess

- Neglect the delayed neutrons and assume all neutrons are prompt.
- Example (fast power increase)
  - let k go from 1 to 1.0005 ($\Delta k$ = 0.5 mk)
  - calculate power rise after 10 s
    - n = 10 000 cycles
  - $P(10s) = P_0\, k^n = P_0\, (1.0005)^{10\,000}$ ♠ 150 $P_0$
  - or, $t = \ell/\Delta k = 0.001/0.0005$ and $e^{10/2}$ ♠ 150
- This is much too fast to control
- A reactor will behave like this on a very large reactivity insertion (Chenobyl)

## Neutron Lifetime - Second Guess

- Include the delayed neutrons to increase the average cycle time
- Approximate their effect assuming all the delayed neutrons have the same lifetime:
  - delayed neutron lifetime = 12.5 s
  - decay constant $\lambda = 0.08\ s^{-1}$ (= 1/12.5 s)
  - delayed neutron fraction is $\beta$ ♠ 0.007 (fresh fuel)
- $\mathcal{L}$ = (0.993 x 0.001 s) + (0.007 x 12.5 s) ♠ 0.1 s
- $\mathcal{L} = (1-\beta) \times \ell + \beta \times (1/\lambda)$ i.e. $\boxed{\mathcal{L} ♠ \beta/\lambda}$

## Power Rise in the Average Lifetime Approximation

- with $\mathcal{L}$ ♠ $\beta/\lambda$ the formula for reactor period becomes

$$\boxed{\tau = \beta/(\lambda\Delta k)}$$

- This is the *average lifetime approximation* because all neutrons are assumed the same, with a common average lifetime.

$$\boxed{P = P_0\, e^{(\lambda\Delta k/\beta)\, t}}$$

- This formula works well for small $\Delta k$ additions typical of reactor control systems.

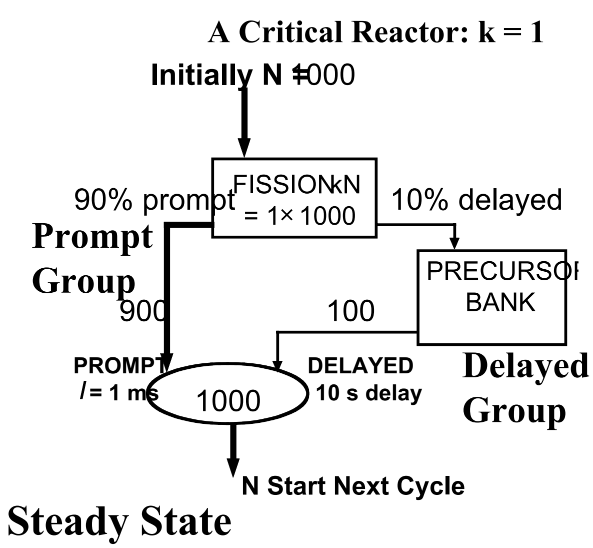
## Example Repeated

- $\Delta k = 0.0005$ ms, $\lambda = 12.5$ s, $\beta = 0.007$ & $t = 10$ s:
  - $\tau = \beta/(\lambda\Delta k) = (0.007 \times 12.5/\ 0.0005) = 175$ s
  - $P(10\text{ s}) = P_0\, e^{10/175} = 1.06\, P_0$
- This is a 6% increase in 10 s for a 0.5 mk reactivity addition, not 15 000% as we saw assuming prompt neutrons alone.
- The rate of increase is less than 0.6% present power per second
- A control system can be designed for this.

## What is Wrong with This?

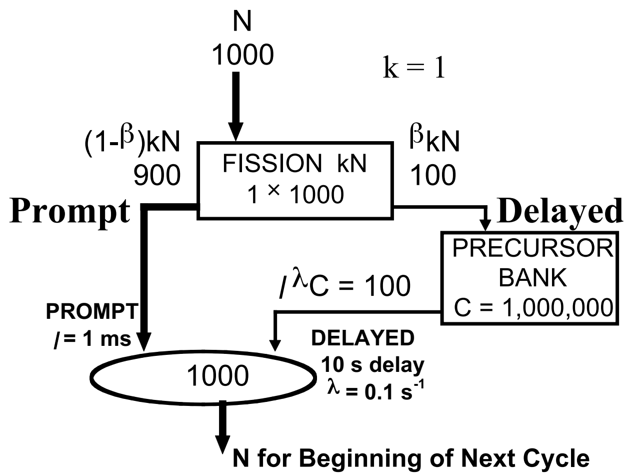
- The delayed neutrons have such an important effect that we must consider their effect more carefully. It is not good enough to just average their lifetime with the prompt neutrons:
- separate the neutrons into two distinct groups, prompt and delayed neutrons, each with its own lifetime.
- This is the *Two Group Approximation*

## Next Guess - Two Groups; all delayed neutrons have the same lifetime

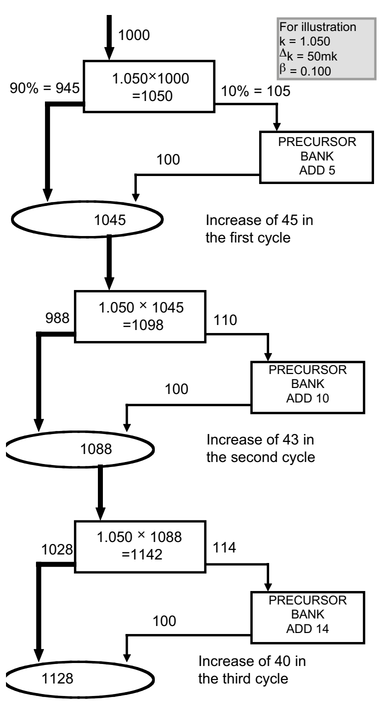



- To Illustrate we use an unrealistic large delayed neutron fraction of $\beta = 10\%$

# A Steady State Critical Reactor




- Here is the same cycle with symbols and artificial numbers




- Here we see the effect through 3 complete cycles after neutron multiplication

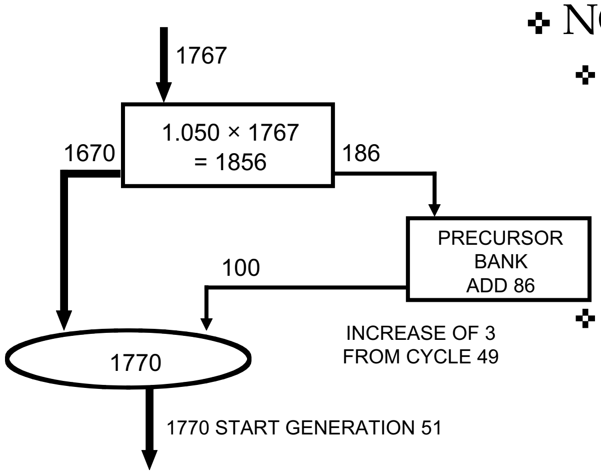
  k = 1 goes to k = 1.050

  $\Delta$k = 50 mk
- This is a completely artificial value for illustration.
  - It is only possible because $\beta$ is also absurdly large.

# What Happens to Each Group?

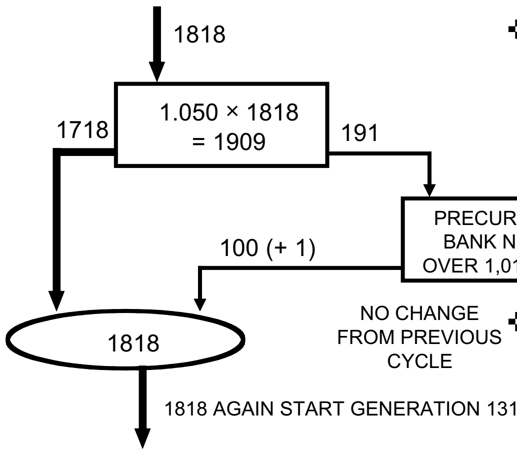
- Notice that the prompt population increases significantly in each cycle,
  - BUT
- The size of the increase is dropping.
- More go into the precursor bank in each cycle.  Initiallythere is no increase in the number coming out.

# Cycle # 50  (about 0.05 s later)



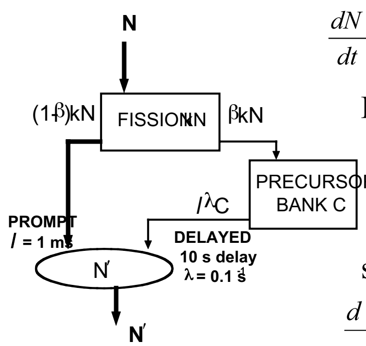

- NOTICE
  - not much increase from cycle 50 to cycle 51
  - large increase in precursor bank each cycle has almost saturated

# Complete Saturation by Cycle 130




- **By now the bank is big enough that it should increase its output.**
- **This now drives power up at a rate controlled by the decay rate.**

# Express a Typical Cycle in Symbols




$$\frac{dN}{dt} \approx \frac{(N'-N)}{\ell} = \frac{(1-\beta)kN}{\ell} - \frac{N}{\ell} + \lambda C$$

Rearranging

$$\left(\frac{1}{N}\right)\frac{dN}{dt} = k\left[\frac{\left(\frac{k-1}{k}\right)-\beta}{\ell}\right] + \frac{\lambda C}{N}$$

so

$$\boxed{\frac{d\ln(N)}{dt} = \left(\frac{1}{N}\right)\frac{dN}{dt} = \left[\frac{\rho-\beta}{\ell}\right] + \frac{\lambda C}{N}}$$

Also

$$\boxed{\frac{dC}{dt} = \frac{\beta kN}{\ell} - \lambda C}$$

**Translate the Symbols into a pair of coupled Differential Equations**

# The Point Reactor Model

- These same equations can be derived by separating the time dependent Diffusion Equation using $\Phi\left(\vec{r}\right) \cdot \phi(t)$
  - a 3-D spatial equation for flux shape
  - an independent time equation
- Many Textbooks derive this

  POINT REACTOR MODEL
- Power changes at each point in the reactor, independent of flux shape.

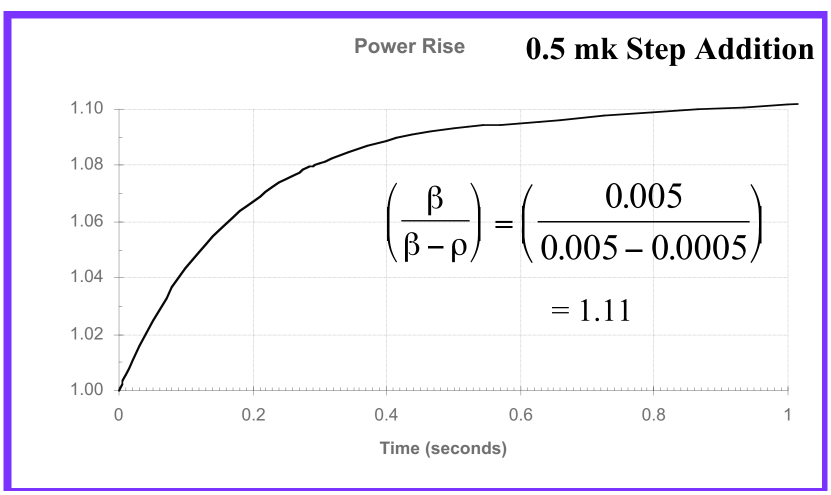
$$\frac{d\ln(N)}{dt} = \left(\frac{1}{N}\right)\frac{dN}{dt} = \left[\frac{\rho - \beta}{\ell}\right] + \frac{\lambda C}{N}$$

- Notice that there are two conditions for which Rate $\mathrm{Log}_e$ N is zero, or nearly zero.
  - Initial Steady State with $\rho = 0$ and $\beta/\ell = \lambda C_0/N_0$
    - i.e. $\lambda \ell C_0$ (decay) = $\beta N_0$ (production)
  - After N has grown from $N_0$ to $[\beta/(\beta-\rho)]N_0$ if C remains at its initial value $C_0$
- In both cases the two terms on the right hand side cancel each other out.

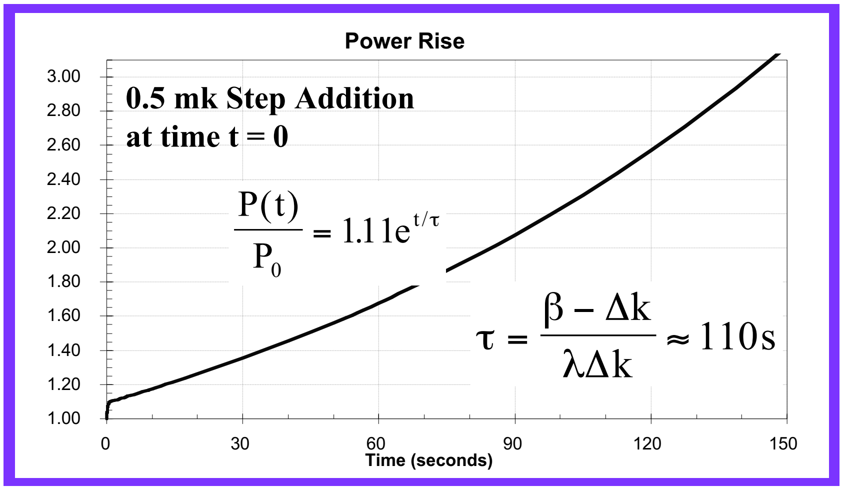
$$\frac{d\ln(N)}{dt} = \left(\frac{1}{N}\right)\frac{dN}{dt} = \left[\frac{\rho-\beta}{\ell}\right] + \frac{\lambda C}{N}$$

- After N reaches the value $\left(\frac{\beta}{\beta-\rho}\right)N_0$ the only way N can grow is for C to grow from its initial value $C_0$
- The Mathematical Solution of the Pair of Equations shows the following:
  - A prompt jump in power of $\left(\frac{\beta}{\beta-\rho}\right)N_0$
  - Followed by a stable exponential rise with a constant rate $\log_e$

# Power after a Step Reactivity Increase
## The Prompt Jump (fast transient)



## Power after a Step Reactivity Increase
### The "Stable" Power Increase




# Mathematical Formula

$$\boxed{P(t) = P_0 \frac{\beta}{\beta - \Delta k} e^{t/\tau} \times \left[1 - \left(\frac{\Delta k}{\beta}\right) e^{-t/\tau_R}\right]}$$

$$\tau = \frac{\beta - \rho}{\lambda \rho} \approx \frac{\beta - \Delta k}{\lambda \Delta k}$$

**is the stable <u>Reactor Period</u>**

$$\frac{1}{\tau_R} = \frac{\beta - \Delta k}{\ell} + \frac{1}{\tau}$$

**$\tau_R$ determines the Transient Rise Time**

# How Good is this Analysis?

- Detailed computer codes are used for Safety Analysis. They include many groups of delayed neutrons, not just a single group. Accurate numerical solutions are possible.
- The two group equation illustrates features of the accurate solutions, and is quite good quantitatively.
  - The initial rise should be even faster,
  - the eventual long term growth a bit slower.

# Dynamics of the Critical Reactor

***Discussion of the Two Group Equation***

***Prompt Criticality***

***Power Rundown***

$$P(t) = P_0 \frac{\beta}{\beta - \Delta k} e^{t/\tau} \times \left[ 1 - \left( \frac{\Delta k}{\beta} \right) e^{-t/\tau_R} \right]$$

- First Simplification
  - The square bracket contributes a factor of 1.0 after the first second, so replace it

$$\boxed{P(t) = P_0 \frac{\beta}{\beta - \Delta k} e^{t/\tau}}$$

$$\tau = \frac{\beta - \Delta k}{\lambda \Delta k}$$

# Simplification for Numerical Work

- It is convenient to rewrite the equation in terms of the ratio $\Delta k/\beta$.

$$P(t) = P_0 \frac{1}{1 - \left( \frac{\Delta k}{\beta} \right)} e^{t/\tau} \qquad \tau = \frac{1 - \left( \frac{\Delta k}{\beta} \right)}{\lambda \left( \frac{\Delta k}{\beta} \right)}$$

- The behaviour depends on the ratio $\Delta k/\beta$, not on the factors independently.

# The Equation for Small Δk

$$P(t) = P_0 \frac{1}{1-\left(\frac{\Delta k}{\beta}\right)} e^{t/\tau} \qquad \tau = \frac{1-\left(\frac{\Delta k}{\beta}\right)}{\lambda\left(\frac{\Delta k}{\beta}\right)}$$

- When Δk << β, the equation becomes:

$$P(t) = P_0 e^{t/\tau} \quad \text{with} \quad \tau = \frac{\beta}{\lambda \Delta k}$$

- This is exactly the result of analysis with the "Average Lifetime Approximation"
  - This supports the statement that the simple formula is good for small reactivity additions.

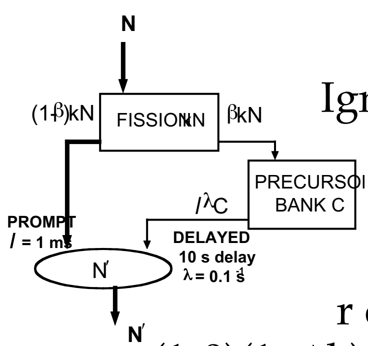
# Normal Control

- CANDU power changes are regulated by the Liquid Zone Control System.
- Flow into 14 water ($H_2O$) compartments is adjusted by inlet valves in response to a power demand.
- Even if this system fails it cannot add more than about 0.1 mk per second max.
- The amount added in a ramp is typically less than 0.01 mk every fi s.

# Normal Control (continued)

- Regulating system design assures that the operators never see a prompt jump.
- The time between step additions on a power ramp allows the prompt jump effect to settle before the next addition.
- Example: $\Delta k$ = 0.1 mk, $\Delta k/\beta$ = 0.0001/0.005 = 0.02
  - $\tau$ = (1-0.02)/(0.1 x 0.02) = 490 s  Jump = 1/(1-0.02) = 1.02
  - 5 minutes later, $P(300)/P_0$ = 1.02 $e^{300/490}$ = 1.02 x 1.84
  - An immediate 2% rise, followed by a steady rise to 1.02 x 1.84 ♠ 1.9 times the initial power, e.g. from 10% F.P. to almost 20% F.P.

# Return to Basics for a Simple Analysis




- Prompt Cycle Alone

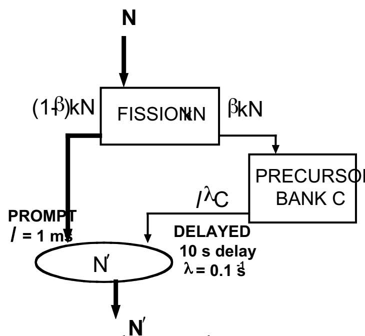
Ignoring Delayed Neutrons

- After n cycles
- $N_n = N_0\ [(1\text{-}\beta)k]^n$
- $N_n/N_0 = r^n$

r defined for convenience

- r = $(1\text{-}\beta)(1+\Delta k)$ ♠ $[1 - (\beta - \Delta k)]$
- For $\Delta k < \beta$, $r < 1$
- As n increases, $r^n$ ∅ 0
- Typical Example: if $\Delta k$ < 1 mk,
- $r^n < 0.01$ for $n > 5000$ (about 5 seconds)

# Effect of Neutrons from the Precursor Bank




- In Steady State, each cycle $\beta k N_0$ neutrons go in and $\beta k N_0$ come out.
- Assume this continues unchanged for a few seconds after $\Delta k$ added.
- Initial cycle $\beta N_0$ (initially k was 1)
- Second cycle $\beta N_0 + r\,\beta N_0$
- Third Cycle $\beta N_0 + r\,\beta N_0 + r^2\,\beta N_0$

# After N cycles (r < 1, so β < Δk)

- nth Cycle $\beta N_0\,(1 + r + r^2 + \ldots + r^n)$
- $S_\forall = \beta N_0(1 + r + r^2 + \ldots + r^n + \ldots$
- $\underline{r\,S_\forall} = \beta N_0( \quad r + r^2 + \ldots + r^n + r^{n+1} + \ldots$

subtract $S_\forall(1-r) = \beta N_0$ so $S_\forall = \beta N_0 / (1-r)$

- $S_n = S_\forall - r^n S_\forall$ so for large n $S_\forall = S_n$
  - and $1-r \spadesuit \beta-\Delta k$
- $$\boxed{P_n = \frac{\beta}{\beta - \Delta k} P_0}$$

# Discussion

- Notice again that there is a rapid *Prompt Jump*.
- The power does not continue to increase past this level unless the growth in the bank is accounted for.
- Adding prompt and delayed together:

$$P_n = r^n P_0 + \frac{\beta}{\beta - \Delta k} P_0 - \frac{r^n \beta}{\beta - \Delta k} P_0$$

$$= \frac{\beta}{\beta - \Delta k} P_0 - \frac{\Delta k}{\beta - \Delta k} r^n P_0$$

## Comparison with 2 Group Equation

- "Simple" analysis showed

$$P_n = \left( \frac{\beta P_0}{\beta - \Delta k} \right) - \left( \frac{\Delta k P_0}{\beta - \Delta k} \right) r^n$$

- The two group equation can be written

$$P(t) = \left( \frac{\beta P_0}{\beta - \Delta k} \right) e^{t/\tau} - \left( \frac{\Delta k P_0}{\beta - \Delta k} \right) e^{-t\frac{(\beta - \Delta k)}{\ell}}$$

- The two group solution shows the effect of allowing for precursor bank growth.

## What Happens if $\Delta k > \beta$ ?

- Now $r = (1 + \Delta k)(1 - \beta) > 1$
- The factor $r^n$, neglected before, now becomes dominant
- The analysis, neglecting the delayed neutrons, looks exactly like the initial simple analysis with all neutrons prompt.
- The condition $(1 + \Delta k)(1 - \beta) = 1$ is called

$\boxed{\text{PROMPT CRITICAL}}$

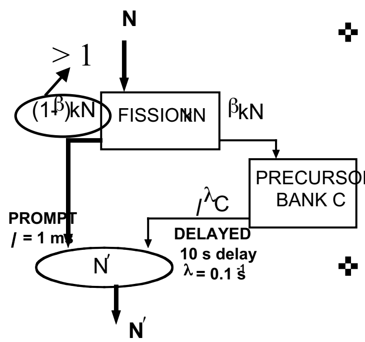
## Prompt Critical

<u>The Condition for Prompt Criticality</u>

- $(1 + \Delta k)(1 - \beta) = 1$
- $\cancel{1} + \Delta k - \beta - \beta\Delta k = \cancel{1}$ so $\Delta k = \beta(1 + \Delta k)$
- $\beta = \Delta k/k = \rho$ or $\beta$ ♠ $\Delta k$
- The reactor is critical even without the delayed neutrons and the power rises at a rate governed by the prompt neutron lifetime ($\ell$ = 1 ms) not d.n. decay ($\lambda$ = 0.1s)

# Why Prompt Critical Must be Avoided

- The prompt population increases out of control even before there is any contribution from delayed neutrons.




- Chernobyl: Initial estimates were + 30 mk added.
- Power rose from 7% to 10,000%in less than 2 seconds

# Equation for Super Prompt Critical

- The two group equation breaks down for Δk ♠ β,
  - detailed analysis that includes more than one group of delayed neutrons gives numerical solutions.
- The equation gives reasonable results for Δk << β **and for** Δk >> β

$$\boxed{P(t) = P_0\left(\frac{\beta}{\beta - \Delta k}\right)e^{t\left(\frac{\lambda \Delta k}{\beta - \Delta k}\right)} - P_0\left(\frac{\Delta k}{\beta - \Delta k}\right)e^{-t\frac{(\beta - \Delta k)}{\ell}}}$$

## Emergency Shutdown

- Normal regulation limits power increase to below about 0.8% present power/s
- When rate of power increase is measured to be 8% present power/s, the regulating system drops absorber rods, a “step back”
- When rate of power increase is measured to be 10% present power/s, Shutdown System #1 activates
- When rate of power increase is measured to be 15% present power/s, Shutdown System #2 activates

## Response: Adding Negative Δk

- The dynamic response of the reactor when devices rapidly insert neutron absorbing material is also quite well modelled by the (simplified) two group equation.

$$\boxed{P(t) = P_0\left(\frac{\beta}{\beta - \Delta k}\right)e^{\left(\frac{\lambda\Delta k}{\beta - \Delta k}\right)t}}$$

- With negative Δk, the prompt jump becomes a prompt drop.

# Example

- When SDS#1, Control Absorber Rods and the Liquid Zones all deploy together they insert nearly - 100 mk
- $\Delta k$ = -0.100, $\beta$ ♠ 0.005, $(\Delta k/\beta)$ = - 20
- $\beta/(\beta-\Delta k) = 1/[1 - (\Delta k/\beta)] = 1/21$ ♠ 0.05
- If initial power was 100% full power there is an immediate drop to 5% of full power.
- The capabilities of the various shutdown systems results in a prompt drop to the range of about 2% to 10%

# Example Continued

- Following the prompt drop, power continues to fall, proportional to: $e^{\left(\frac{\lambda \Delta k}{\beta - \Delta k}\right)t}$
- Notice that for very large negative $\Delta k$, such as used for emergency shut down,

  $|\Delta k| >> \beta$ , giving $e^{-\lambda t}$
- In the actual rundown, decay of delayed neutrons with different lifetimes results in $\lambda$ changing gradually to a smaller number